European
Commission

Pre-retirement saving and the extension of working life

A model based assessment of the impact of low interest rates

Working Paper 2/2015

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During the aftermath of the financial crisis, certain paradoxical trends have emerged in Europe. Firstly, despite the context of economic adjustment and restructuring, the employment rate of older workers has increased in most countries, and secondly, saving rates have remained remarkably resilient to the interest rate squeeze pursued by central banks as an economic stimulus. The question arises, whether lower interest rates effectively discourage or rather encourage saving among older workers, or even constitute an incentive to work longer, in case their saving strategy aims at maintaining a standard of living after retirement. The working paper adresses this issue through a model based approach.

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Cataloguing data as well as an abstract can be found at the end of this publication.

Luxembourg: Publications Office of the European Union, 2015

ISBN 978-92-79-46204-7
ISSN 1977-4125
doi: 10.2767/78245

Pre-retirement saving and the extension of working life: a model based assessment of the impact of low interest rates [1]

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[1] Acknowledgement: the author, who worked until 2014 as an official of the European Commission (notably in the Directorate General for Employment and Social Affairs) benefited from technical support from Dr. Jörg Peschner of the latter department. He is also indebted to the Commission's Reading Committee for their helpful comments.

Summary

During the aftermath of the financial crisis, certain paradoxical trends have emerged in Europe. Firstly, despite the context of economic adjustment and restructuring, the employment rate of older workers has increased in most countries, and secondly, saving rates have remained remarkably resilient to the interest rate squeeze pursued by central banks as an economic stimulus. Anecdotical evidence reveals a particular concern among older workers about the impact of the financial crisis on their savings, which play a role in their retirement planning.

This paper illustrates, on the basis of an inter-temporal actuarial model, the likely impact of low interest rates on the economic behaviour of older workers (aged over 50). The model can be seen as a simplified version of premium based pension plans, for which it offers a first indication of possible under-financing when interest rates decline sharply. It also yields fairly constant elasticities which can be exploited for wider modelling purposes. The paper finds that, under the model's assumptions, lower interest rates may contribute to a substantial postponement of the retirement age, but also tend to invigorate saving in order to sustain standards of living after retirement. While the first response is in line with policies to address the demographic challenge and finance social protection, the latter depresses consumption and may therefore be counter-productive for the economic recovery. The paper also illustrates, on the basis of national key indicators, how the balance between the two impacts depends upon the demographic and labour market characteristics proper to each country.

1. Introduction

The strategy of lowering key interest rates in a protracted way, pursued by central banks to provide stimulus for economic recovery, has radically changed the economic context in which older workers envisage retirement. Active people approaching retirement may be particularly concerned by the unattractive saving opportunities[2]. As they tend to have a significant part of their income available[3], with mortgages and family expenses weighing less heavily, savings are often a direct element of their retirement planning[4].

Pre-retirement saving can cushion the expected income loss when moving from labour income to statutory ("pay as you go") and corporate pension(s) and is therefore actively promoted in some countries i.a. by fiscal means. However, due to the persistently low interest rates and returns on investments, the funding of these schemes is jeopardized and regulatory constraints (such as minimum returns) questioned. Still, the intrinsic long term impact of low interest rates remains somewhat unexplored[5], apart from the risk of real estate bubbles[6].

[2] Between 2009 and 2014, the interest rate declined in the EU on average from around 4 to nearly 2%, and the real interest rate (corrected for inflation) even sharper from 4 to 1,5 %.

[3] according to latest data of SILC for 2013, the median income in the age group 50-64 in the EU was around 6% higher than for the age group 25-49.

[4] see Cantor and Yuengert, *The baby boom generation and aggregate saving*, p. 78

[5] E.g. in the CFA book "*Life annuities, an optimal product for retirement income*", the effect of duration is said to be a good proxy for the sensitiveness of life annuities to interest rates (p. 54).

[6] in a recent CEPS paper, Brender, Pisani and Gagna point to real estate bubbles as a frequent side effect of low interest rates (p. 38 ff.).

To ilustrate the impact of low interest rates on the behaviour of the 50-64 age group, we consider the case of an individual making additional savings as of the age of 50 in view of topping up his statutory and corporate pension rights (i.e. complementing the first and second pension pillar by a so-called "third pension pillar"). In section 2, a basic actuarial model balancing long-term savings and cash withdrawals is presented. In section 3 the retirement age is introduced as the variable which smooths the impact of an interest rate change, while in section 4 a dynamic model is presented, in which an intermediate interest rate change occurs. Section 5 discusses the limitations and relevance of the model, and section 6 presents national simulations. In section 7 the relevance of elasticities for modelling is shortly discussed, and conclusions are drawn in section 8.

2. Model with fixed retirement age

In the model analysed below,
T denotes the expected remaining life-time at 50[7]
w the years of work remaining until retirement or withdrawal from the labour market
s the annual savings during w years in view of topping up the expected acquired pension(s)
p the annual planned withdrawals from the capital saved before retirement
i the interest rate on savings (or return on investment), held constant during years of work (w) and retirement (T-w) and ***r*** the discount rate corresponding to i, where

$$r = 1/(1+i) \qquad [1]$$

In the simple version of the model, with constant annual savings cc. withdrawals, the model is spelled out in terms of cash flows and the appropriate interest rate is the nominal one. A slight re-specification of the model, whereby the cash flows grow with inflation, yields similar formulas expressed in terms of the real interest rate (see Technical Annex section A) .

Below a formula is derived for the *"funding ratio"* ***s/p***, i.e. the annual savings (s) required to obtain an annual additional revenue (p) as of the first year of retirement. The value of p can be expressed as an equivalent of pre-retirement income, and depend upon acquired pension rights under statutory and corporate pension schemes (as well as the amount of capital already acquired before the age of 50[8]).

At the age of retirement, the annual savings s during w years will have generated a capital C

$$C = s\left[\frac{(1+i)^{w}-1}{i}\right] \qquad [2]$$

At the age of retirement, the capital C should cover the present value of all annual withdrawals p during T - w years:

$$C = \frac{p}{i}\left[1-\left(\frac{1}{1+i}\right)^{T-w}\right] \qquad [3]$$

[7] Both T and w are calculated as of the age of 50, but alternative reference ages could be used for convenience.
[8] It is assumed that such capital does not otherwise affect the model than through the setting of the level of the post-retirement supplementary income p. Alternatively, capital existing at the the age of 50 could be transformed into "backdated savings", and the planning period accordingly reduced below 50.

Consequently, with constant interest rate i (and the same goes for the discount rate r) the balance[9] between savings and withdrawals is met if

$$\frac{s}{p} = r^{w}\left(\frac{1-r^{T-w}}{1-r^{w}}\right) \qquad [4]$$

The above formula grasps the combined effect of assets building up slower as the interest rate diminishes, together with a diminishing potential for withdrawals from the composed capital after retirement (due to lower interest on the remaining capital). Both effects are separately illustrated by Tables 1-A and 1 - B.

Table 1-A: values of C/s with diminishing i

i in %	w = 15	w = 11,5
5	21,6	15,1
4	20,0	14,2
3	18,6	13,5
2	17,3	12,8
1	16,1	12,1
0	15,0	11,5

Table 1-B: values of p/C with diminishing i

i in %	T-w = 15	T-w = 20,7
5	0,096	0,079
4	0,090	0,072
3	0,084	0,066
2	0,078	0,059
1	0,072	0,054
0	0,066	0,048

As a result and shown in Table 1-C, the funding ratio (s/p) increases with diminishing interest i, decreasing contribution or savings period w and lengthening withdrawal period T - w.

Table 1-C. Values of s/p for various combinations of i, T and w [10]

i	*T = 30*	*T = 30*	*T = 32,2*
in%	*w = 15*	*w = 11,5*	*w = 11,5*
5	0,481	0,790	0,845
4	0,555	0,905	0,975
3	0,642	1,040	1,130
2	0,743	1,199	1,315
1	0,861	1,387	1,535
0	1,000	1,609	1,800

Under certain assumptions, the complex formula for the elasticity of the savings intensity s/p to r is reduced to the simple expression $\varepsilon(s/p,r) = w$ (see Technical Annex Section D). This suggests that, for the response to structural changes in the interest rate, the funding ratio and the age of retirement are interchangeable variables. Therefore we will examine hereafter a variant of the model, allowing to change the retirement age instead of s/p.

[9] It is possible to avoid exhaustion of the capital at the end of the expected lifetime, by introducing a restriction on the share of C which is to be consumed. This can be done by dividing the right hand side of equation [3] by the envisaged share for consumption during retirement.
[10] average life expectancy and exit age after 50 for EU27 are resp. 32,2 and 11,5; see also section 6.

3. Model with variable retirement age

Recalling equation [4] and considering s/p as fixed, the retirement age can be written as

$$w = \ln\left(\frac{\frac{s}{p}+r^{T}}{\frac{s}{p}+1}\right)/\ln r \qquad [5]$$

The Technical Annex (Section B) sets out how equation [5] is derived from [4].

As Table 2 suggests, the required working or saving time increases by nearly 10 years for quadrupling of the ratio s/p, and by more than 3 years for any value of s/p when the interest rate decreases from 4 to 1 %.

Table 2: values of w at various levels of s/p for T = 32,2[11]

i	*s/p*		
in %	*1*	*0,5*	*0,25*
5	10,3	15,4	20,6
4	11,3	16,6	21,7
3	12,4	17,8	22,9
2	13,6	19,1	23,9
1	14,8	20,3	24,9
0	16,1	21,5	25,8

It should, however, be acknowledged that working longer normally generates additional pension rights, so that for a pre-determined funding ratio s/p, the denominator tends to become dependent on w. A calculation of a stable solution for w through iteration is proposed in the Technical Annex (Section E) and applied in Table 4.

4. Dynamic formulation of the model

An individual might be faced with a change in interest rate at other ages than just 50. In the dynamic formulation of the model, presented in Section F of the Technical Annex, a sudden and permanent reduction of the interest rate in a particular year n between the age of 50 and the moment of retirement is considered. This affects the accrual of the capital built up by savings as of year ***n***, as well as the present value of the withdrawals at the time of retirement.

At the moment of the interest rate change, the individual disposes of two options allowing to maintain the level of annual withdrawals after retirement:
- increasing the savings (or funding ratio as p is given) as of year n+1 while keeping the retirement age constant, or
- postponing retirement while keeping the annual savings unchanged.

[11] average life expectancy at 50 for the EU

Table 3 summarizes the results obtained with an interest rate reduction from 4% to 1%[12] at various moments, using the European average exit age and life expectancy as reference. It can be seen that, the later the interest rate change occurs, the sharper the adjustment needed in terms of savings. The adjustment in terms of retirement age, however, declines slowly from 3.5 years extra at age 50 to 2.7 years extra at age 60. The postponement of retirement becomes eventually the only feasible option in later years, as an adjustment in terms of saving becomes prohibitive towards the age of 60 (at which there would be a quadrupling of the saving required).

The second column of Table 3 allows an estimation of the (theoretical) degree of under-funding of premium-based pension systems, if the premium cannot be increased and other commitments (retirement age) remain unchanged for the pension system. That is, if the insured person is aged 55, the shortfall of contributions should be 99% of an annual premium during the remaining 6 years until retirement, i.e. around 6 full premiums. In that situation, unless the insured person accepts reduction of pension withdrawals later on, the only option for the insurer would be to generate more income by seeking a higher return on invested premiums (usually meaning a higher risk).

Table 3: impact of interest rate reduction from 4% to 1% at age n (assuming T = 32,2 and w=11,5)

Age (n) of reduction	*Ratio (in %) between savings after and before n*	*Number of years [13] postponement of retirement*
50	157	3,5
51	163	3,5
52	170	3,5
53	177	3,5
54	187	3,4
55	199	3,3
56	215	3,2
57	236	3,1
58	269	3,0
59	327	2,9
60	458	2,7

The appendix presents some simulations of the potential degree of under-funding of pension plans.

5. Limitations and relevance of the model

Although simple in presentation, the model constitutes a relevant framework for analysis of a more complex and realistic economic context. Notwithstanding the fact that it focuses on the third pension pillar, it is relevant for the second pillar and may be helpful for assessing the interaction with first pillar reforms.

[12] This scenario can be compared to the recent trend in interest rates.
[13] It should be noted that the results at age 50 are consistent with those in Tables 1C and 2 (for an exact match at the proper values of s/p and/or w see Col. F and G of Table 4). They can also replicated at country level.

The model assumes the perception by rationally behaving older workers of a long-term stability of interest rates. This is justified by the current temporal structure of interest rates with both very low short- and long-term rates.

The model considers investments in the form of bank deposits and/or bonds by a risk avert saver, which is the prevailing type according to available enquiries[14]. However, the model can also be applied to more speculative investment strategies e.g. investment in stocks; in that case, the return on investment expected by the investor only differs from the market interest rate by a risk premium[15].

Long term savings are predominantly made in the form of investment or pension plans managed by financial institutions, with an agreed risk profile. When considered for a large group of individuals with the same characteristics (age, contribution period), the model can be seen as a simplified version of premium based pension plans. Compared to the fixed withdrawals proper to the model, such plans imply a redistribution within the group to those surviving the life expectancy.

The reference age of 50 is conventionally chosen because 50 can be seen as a milestone in one's career and lifetime; this parameter can be varied a few years up- or downwards without altering the main conclusions of the analysis[16]. The choice of 50 is, however, consistent with the calculation method of Eurostat's exit age indicator, which is based on exit probabilities between the ages of 50 and 70.

6. Quantification for EU Member States

The quantification for EU Member States may help to illustrate the possible responsiveness of the retirement age and the funding ratio, respectively, to changes in the interest rate. It was carried out in terms of comparative statics using formulas [3] and [4].

Some limitations affect the validity of intra-country comparisons. In particular, it should be noted that differences in replacement rates of pensions between countries[17] may imply, together with uncertainties over e.g. statutory pension reforms, differences in the desired level of complement to pensions (cf. parameter p). In addition, the savings potential may to a different extent be limited by income constraints. However, as the central variable in the model is the ratio between savings and pension complements (s/p), any working value of the ratio should in principle be deemed compatible with the national situation and suitable for a sensitivity analysis.

The following statistical series were used (totals for men and women) :
- for variable w, the national values stem from the average exit age from the labour force[18] as published by Eurostat, reduced by 50 (which is the conventional starting age of the model);

[14] for example, the share of Germans unwilling to invest in riskier assets than bank deposits rose to 67% from 63% in late 2013, according to a survey conducted by GfK in late 2014 (reported by FAZ).
[15] see the theory of finance developed by E. Fama and M. Miller, 1972. With a constant risk premium of e.g. 2%, an interest rate decline from 4% to 1% would correspond to a return rate decrease from 6% to 3% and vice versa.
[16] For example, setting the reference age at 45 would amplify the results of col. F and I in Table 4 by around a quarter.
[17] For details see the EU report on adequacy of pensions, 2012 or OECD.
[18] variable lfsi_exi_a, data for 2010 or when unavailable the last year available (2009 for EL, NL, FI, UK, EE, HR, CY and SK; 2008 for LV; 2007 for BE, IE, AT, PL and PT; 2006 for BG, LT, PO and SI; 2005 for LU). It should be acknowledged that Eurostat has discontinued the publication of this indicator for quality reasons, but there is no alternative indicator available which conceptually fits in the model.

- for variable T, the national values stem from the life expectancy at 50 as published by Eurostat, reduced by 50 as for the previous variable[19].

The key context indicators (exit age, life expectancy) and the results of the sensitivity analysis are summarized in Table 4.

As to the scope for extending working life, the average extension of the exit age resulting from a decrease of the interest rate from 4 to 1% would be 2.8 years in the EU27[20](see col. I) . However, the effects are not commensurate with the employment performance of older workers in the respective countries. Countries where the interest squeeze would lead to an extension of working life above the EU average include Germany, Ireland, Spain, Portugal, and Cyprus, but also the Netherlands, Sweden and the UK, where employment rates for older workers are among the highest. Conversely, the effect is below average in Luxembourg (where the employment rate of older workers is only 40%) and most new Member States. However, direct comparisons between Member States should be made with caution. Considering for example four countries with an identical remaining life expectancy of 32,7 but different average exit ages at the age of 50 (Austria 10,9: UK 13; Netherlands 13,5 and Ireland 14,1), the extension of working life would be higher in the Netherlands and Ireland (3 years), than in Austria (2,8) and UK (2,9), which may seem counter-intuitive. This has to do with the model's higher implicit savings intensity (funding ratio) in the latter two countries - as a result of their lower initial average exit age -, making the effect of an additional year of contribution higher than in the two other countries with lower savings intensity.

There are also striking differences across countries in terms of response of the funding ratio to a decline in interest rate from 4 to 1% (see col. F). On average in the EU27, diminishing the interest rate from 4% to 1% leads to a savings increase by more than half. Above-average increases are observed in most EU-15 Member States; conversely, the increases are below average in the new Member States (except Malta and Cyprus). Overall, lower interest rates would trigger a stronger saving reaction in those Member States where older workers happen to dispose of the highest incomes and purchase power. The demographic effect seems to be the predominant factor underlying these results. This is well illustrated by comparing the cases of Denmark and Spain, which have identical exit ages (12,3 years), whereas life expectancies at 50 differ consederably (Denmark 31,6 vs Spain 34). Since the retirement period is consistently longer than the active period after 50, a lower interest rate has a relatively strong effect on the required funding during the pre-retirement years, which explains why the funding ratio shows an increase by nearly 5 points more in Spain than in Denmark (+61,4% vs +56,7%).

7. Elasticities and their contribution to labour market modelling

From what preceeds, the impact of the interest rate reduction by 3% is proportionally higher in terms of savings (more than 50%) than in terms of working life extension (2,8 years or less than 25% of the initial 11,5 years before exit). This is also confirmed by the typical values of elasticities (see Technical Annex Section D), according to which changes in interest rates trigger stronger relative changes in terms of savings than in terms of postponing retirement (the elasticity values are concentrated around

[19] data for 2012 extracted from Eurostat's demography database, variable "life expectancy by age and sex" (demo_mlexpec)
[20] The EU aggregate is for EU27, as the exit age statistics were discontinued by Eurostat and an agregate is not calculated for EU28

15 and 10 respectively, over the whole interest rate range and across Member States).

This could inspire a reconsideration of the function governing the retirement decision in models like the Commission's Labour Market Model (LMM)[21]. In the LMM, the retirement decision is essentially dependent upon the implicit tax rate on continued work, as presented by Duval in 2003, which exclusively looks at the costs and benefits of "pay as you go" pension systems. The ratio between the elasticities obtained with the present model (10/15 or 0.66) can best be compared - taking account of the fact that also the post-retirement period is considered - with the results obtained by Börsch-Supan in 2000, which amounts to an elasticity of retirement with respect to benefits equal to 0.93[22].

8. Conclusions

This study aimed to provide, on the basis of an analytical model applied to actual labour market, demographic and financial data, quantified scenarios for the intrinsic impact of lasting low interest rates on savings and retirement decisions of older workers. The model can also be used to help in the assessment of the effects on pensions, especially premium-based schemes, and fine-tune the retirement decision function in modelling the labour market.

In simple terms, the simulations show that someone at the age of 50, wishing to maintain the future revenue from pre-retirement savings and faced with a 3% drop in interest rates, has the choice between either working between 2 and 3 years longer, or increasing savings by half until retirement (or a combination of both). The dynamic version of the model suggests that the postponement of retirement becomes the most feasible outcome the later in the career the drop in interest rates occurs. By extension, these significant findings illustrate the challenge for financial institutions to honour fixed commitments or mandatory returns in pension plans without taking additional investment risks.

Although the discouragement of early retirement is not the primarily intended effect of low interest rates, there could be a significant effect in terms of postponement of retirement according to national simulations with this model. However, the effect seems not always strongest in countries where the need to extend working life is most pressing. It would also appear that the effect on pre-retirement savings is most significant, but concentrated in countries with the highest consumption potential, thus constraining the recovery on the consumption side.

Which one of the two responses (extending working life or increase savings) would prevail in reality, remains an open question and would require more surveys. The results delivered by the model suggest a stronger sensitivity in terms of savings than in terms of postponing retirement. But lower interest rates could certainly reinforce active ageing policies pursued with other measures, notably pension reforms.

[21] cf. Employment and Social Developments in Europe reviews since 2011.
[22] see documentation of the LMM part II, p. 73

Table 4. Summary of national results

			:	*for i*	*= 4%*	:	*for i*	*= 1%*	:	*for*	*i= 1%*	*and s/p°*
Coun-try	*w°*	*T*	:	*s/p°*	*εs/p,r*	:	*s/p*	*Δ s/p %*	:	*initial w*	*final w**	*Δ w*-w°*
Col.	*A*	*B*	:	*C*	*D*	:	*E*	*F=E-C*	:	*G*	*H*	*I=H-A*
			:			:			:			
BE	11,6	32,3	:	0,965	15,2	:	1,521	***57,7***	:	15,1	14,4	**2,8**
DK	12,3	31,6	:	0,856	15,1	:	1,342	56,7	:	15,8	15,0	2,7
DE	12,4	32,5	:	0,871	15,4	:	1,380	***58,5***	:	16,1	15,3	**2,9**
IE	14,1	32,7	:	0,701	15,9	:	1,122	***60,0***	:	17,9	17,1	**3,0**
EL	11,5	32,5	:	0,985	15,3	:	1,555	***58,0***	:	15,1	14,3	**2,8**
ES	12,3	34,0	:	0,924	16,0	:	1,492	***61,4***	:	16,2	15,3	**3,0**
F	10,2	34,0	:	1,234	15,5	:	1,974	***60,0***	:	13,8	13,0	**2,8**
I	10,4	33,7	:	1,189	15,5	:	1,898	***59,6***	:	14,0	13,2	**2,8**
L	9,4	32,9	:	1,351	15,0	:	2,127	***57,4***	:	12,7	12,0	2,6
NL	13,5	32,7	:	0,758	15,7	:	1,210	***59,6***	:	17,3	16,5	**3,0**
AT	10,9	32,7	:	1,077	15,2	:	1,702	***58,0***	:	14,4	13,7	**2,8**
PT	12,6	32,4	:	0,845	15,4	:	1,339	***58,5***	:	16,3	15,5	**2,9**
SF	11,7	32,6	:	0,961	15,3	:	1,521	***58,3***	:	15,3	14,5	**2,8**
S	14,4	33,2	:	0,687	16,1	:	1,107	***61,2***	:	18,3	17,5	**3,1**
UK	13	32,7	:	0,809	15,6	:	1,289	***59,3***	:	16,7	15,9	**2,9**
BG	14,1	27,2	:	0,544	13,7	:	0,811	49,1	:	16,8	16,3	2,2
CY	12,8	32,7	:	0,831	15,6	:	1,323	***59,2***	:	16,5	15,7	**2,9**
CZ	10,5	29,9	:	1,046	14,1	:	1,594	52,5	:	13,5	12,9	2,4
EE	12,6	29,7	:	0,765	14,4	:	1,171	53,2	:	15,7	15,1	2,5
H	9,7	27,6	:	1,090	13,1	:	1,610	47,8	:	12,3	11,8	2,1
HR	10,6	29,2	:	1,005	13,8	:	1,519	51,2	:	13,5	12,9	2,3
LV	12,7	27,6	:	0,685	13,6	:	1,023	49,2	:	15,4	14,9	2,2
LT	9,9	27,9	:	1,067	13,2	:	1,584	48,4	:	12,5	12,0	2,1
MT	10,5	32,5	:	1,134	15,0	:	1,785	***57,4***	:	13,9	13,2	2,7
PL	9,3	29,4	:	1,239	13,7	:	1,870	50,9	:	12,1	11,5	2,2
RO	14,3	27,6	:	0,540	13,9	:	0,811	50,0	:	17,0	16,5	2,2
SI	9,8	31,8	:	1,233	14,7	:	1,919	55,6	:	13,0	12,3	2,5
SK	8,8	28,6	:	1,310	13,3	:	1,954	49,2	:	11,4	10,9	2,1
			:			:			:			
EU27	11,5	32,2	:	0,975	15,1	:	1,535	***57,4***	:	15,0	14,3	**2,8**

Notes to table 4

- w° and T are the average exit age and life expectancy counted from the age of 50;
- col. C and E present country specific results comparable to those in Table 1-C;
- in col. F and I the countries with a result equal or above the EU average are marked in bold;
- elasticities (see col. D) are point elasticities calculated following formulas in Technical Annex Section D;
- initial w values in col. G are calculated in a similar way as the values in Table 2 (using the s/p° values of col. C where i is set at 0.4%);
- w* (see col. I) is calculated according to the procedure set out in the Technical Annex section E.

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Technical Annex

A. Basic equations.

Equations [2] and [3] - future resp. present value of regular annual cash flows - are well documented elements of financial calculus,[23] and formula [4] is obtained by divis-iding them.
If s and p grow with a constant inflation rate g, the cash flows follow the pattern

- s in the first year, (1 + g)s in the second year, $(1 + g)^2$ s in the third year etc.

- $(1 + g)^w$ p in the first year of withdrawal, $(1 + g)^{w+1}$ p in the second year etc.

In this case formula [4] and [5] remain identical, but r should be read as the real dis-count rate (1 + g) / (1 + i).

B. Model solution with variable retirement age

Equation [4] can be re-written as

$$r^w = \frac{r^T + s/p}{1 + s/p} \qquad [4b]$$

Substituting r^w by $e^{w \ln r}$ and logarithmizing, one obtains

$$w = \frac{\ln\left[\frac{r^T + s/p}{1 + s/p}\right]}{\ln r} \qquad [5]$$

C. The special case of a zero-interest rate

Although equations [4] and [5] are undefined for i = 0 and r = 1, it can be demon-strated[24] that, when the interest rate tends to zero, the funding ratio (s/p) has (T - w) / w as limit value:

$$\lim_{i \to 0} s/p = \frac{T - w}{w} \qquad [4°]$$

[23] Basic equations [2] and [3] can for example be found in *http://www.tvmcalcs.com/tvm/formulas/regular_annuity_formulas* ("future value / present value"); similar equations for cash flows, which increase with a constant inflation rate, can be found in *http://www.financeformulas.net/General-Finance-Formulas.html* ("growing annuity - future / present value of growing annuity"). It should be noted that both savings and withdrawals are considered to take place at the end of the respective years.
[24] by applying "de l'Hôpital's rule" lim f(x)/g(x) = lim f'(x)/lim g'(x) to equation [4].

Referring to Table 1-C, the value of s/p corresponding to T = 30, w = 15 and i = 0 would be 1, meaning that, without interest accumulation, the savings of 15 years just cover equivalent withdrawals during the remaining 15 years.

The "mirror" equation of [4°] presenting the limit value of w would then read as follows:

$$\lim_{i \to 0} w = \frac{T}{1 + (s/p)} \qquad [5°]$$

D. Use of elasticities

The elasticity of s/p resp. w with respect to r expresses the relative or percentage change in these variables, resulting from a percentage change in r, i.e.

ε(s/p,r) = [d(s/p)/dr] / [(s/p) / r] = [d(s/p)/s/p] / [dr / r]

resp.

ε(w,r) = [dw/dr] / [w / r] = [dw/w] / [dr / r]

and can be expressed as

$$\varepsilon_{s/p,r} = \frac{w \cdot r^{w}\left(1-r^{T}\right) - T \cdot r^{T}\left(1-r^{w}\right)}{\left(1-r^{w}\right)\left(r^{w}-r^{T}\right)} \qquad [4a]$$

$$\varepsilon_{w,r} = \frac{T \cdot r^{T}}{\left(r^{T}+s/p\right) \cdot \ln\left(\frac{r^{T}+s/p}{1+s/p}\right)} - \frac{1}{\ln r} \qquad [5a]$$

A particular feature of [4a] is that Ɛ(s/p,r) = w if w = T/2

Working with elasticities presents some advantages:
- the elasticity is fairly constant across the range of variables i or r, T and w. The value of ε(s/p,r) is typically around 15[25] , whereas the value of ε(w,r) is typically around 10[26];
- each ppt decline in i means a ppt increase in dr/r : e.g. if i is changed from 4 to 2, dr/r nearly doubles (x 1,96);
- consequently, as a rule of thumb, a ppt decline in i triggers an increase of around 15% of s/p and of around 10% of w (or, applied to Table 1-C each row approximately equals the upper row marked up by 15%, and each row of Table 2 equals approximately the upper row marked up by 10%).

E. Iterative method to capture the interaction with pension rights

For an individual, variations in w may induce financial impacts outside the scope of the model. Indeed, once (s)he is pushed by changes in interest rate to extend working life, there may be additional pension rights built up which were not initially counted

[25] with a spread of 3.5 (see Table 3 col. D)
[26] with a spread of 5.4

for. These constitute an external benefit on top of the targeted p, and consequently part of p could be made up by these extra pension rights instead of savings.
To take this into account, one should recalculate w for a value of s/p, in which the targeted p is lowered in accordance with the additional pension rights. The thus revised value of w implies in turn additional pension rights, to which s/p should be adapted. This can be repeated until a stable value of w is achieved.
In our examples, where it was assumed that each additional year of contribution above the average exit age of the country yields a 3,3% bonus on w/p, a stable solution was generally found after 3 or 4 runs.
In the example of the EU27 presented in table 4 (col. I), the runs were as follows:

1. initial value of w obtained with s/p = 0,975: w = 15 or 3,5 more than average exit age
2. second estimate of w obtained with s/p = 1,103[27]: w = 14,0 (2,5 more than av. exit age)
3. third estimate of w obtained with s/p = 1,064[28] : w = 14,3 (2,8 more than av. exit age)
4. fourth estimate of w obtained with s/p = 1,076[29]: w = 14,2

the latter being a stable and final solution[30].

F. Dynamic model formulation

Consider a situation where an individual plans to retire after w years, and to draw an additional income p during T-w years for the rest of his lifetime by depleting a capital built up by saving annually an amount s° during the active period.
Initially, the prevailing long term interest rate was i°, and thus the funding intensity was set at

$$S_0 = P \cdot r_0^{w} \left(\frac{1-r_0^{T-w}}{1-r_0^{w}}\right) \qquad [4]$$

Assuming that, after n years, the long term interest rate shifts to i, the life-time cashflow equation, derived from formulas [2] and [3] reads as

$$S_0\left[\frac{(1+i_0)^{n}-1}{i_0}\right](1+i)^{w'-n} + S'\left[\frac{(1+i)^{w'-n}-1}{i}\right] = \frac{P}{i}\left[1-\left(\frac{1}{1+i}\right)^{T-w'}\right] \qquad [6]$$

in which the first and second term of the left hand side of the equation represent, respectively, the value of the savings before and after year n at the moment of retirement, and the right hand side the present value of withdrawals at the moment of retirement.

The model can be analysed using two options:
1) adapting the savings to s' while keeping the working/contribution period constant (w' = w)

[27] 0,975 / (1 - (0,033 * 3,5)
[28] 0,975 / (1 - (0,033 * 2,5)
[29] 0,975 / (1 - (0,033 * 2,8)
[30] all further runs lead to changes after the first decimal

2) adapting the working/contribution period to w' while keeping the savings period constant (s' = s°)

The solution of w' resp. s' is a function of parameters i°, i, T, w and n. More in particular,

$$\frac{s'}{s_0} = \left[\frac{(1-r_0^{w})(1-r^{T-w})}{(r^{n-w}-1)(r_0^{w}-r_0^{T})}\right] + \left[\frac{r_0(1-r)(1-r_0^{-n})}{r(1-r_0)(1-r^{w-n})}\right] \quad [7]$$

The solution of w' can further be corrected to take into account the interaction with pension rights using the method set out in Section E.

Appendix

Theoretical under-funding of pension plans

The results below illustrate the theoretical extent of under-funding of pension plans on the basis of the following assumptions:

- the calculation is made for pension plans where both *contributions and benefits are defined* in advance and cannot be adjusted in reaction to an interest rate shock;
- the interest rate shock (a squeeze from 4% to 2,5% cc. 1%), limiting the return on investment of premiums, *takes place at half-way the contribution period*;
- the contribution period is varied with a start between 25 and 55, with five year intervals and ending at 65*;
- under-funding is calculated as described in the last paragraph of Section 4, i.e. as the ratio between the missing premiums ("shortfall of contributions"), and the equivalent of annual premiums required to maintain the actuarial balance after an interest rate shock (total of actual number of premiums and missing ones);
- the life expectancy remaining at the start of contributions is based on Eurostat statistics.

Theoretical under-funding under different scenarios (in %)

Contribution ending at 65 and starting at age	Life expectancy remaining at start	Age at which interest shock occurs	Size of interest rate shock (at mid-contribution period)	
			from 4% to 2,5%	from 4% to 1%
25	56,0	45	38,6	58,3
30	51,1	47,5	34,7	54,1
35	46,3	50	30,9	49,7
40	41,5	52,5	27,2	45,1
45	36,7	55	23,6	40,5
50	32,2	57,5	20,4	36,0
55	27,3	60	17,4	31,6

* For methodological reasons, the average exit age is unsuitable for simulations using a starting period before the age of 50.

KE-EW-15-002-EN-N

European Commission

Pre-retirement saving and the extension of working life: a model based assessment of the impact of low interest rates

Luxembourg: Publications Office of the European Union, 2015

2015 — 18 pp. — 21 × 29.7 cm

ISBN 978-92-79-46204-7
ISSN 1977-4125
doi: 10.2767/78245

This publication is available in electronic format in English.