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A Tradeoff between the Output and Current Account Effects of Pension Reform

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A Tradeoff between the Output and Current Account Effects of Pension Reform

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Abstract

We compare the long-term output and current account effects of pension reforms that increase the retirement age with those of reforms that cut pension benefits, conditional on reforms achieving similar fiscal targets. We show the presence of a policy trade-off. Pension reforms that increase the retirement age have a large positive effect on output, but a small (and often negative) effect on the current account. In contrast, reforms that cut pension benefits improve the current account balance but reduce output. Mixed pension reforms, which extend the working life *and* cut pension benefits, can simultaneously boost output and the current account.

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I. INTRODUCTION

Economies in the periphery of the euro area face (at least) three major policy challenges: to preserve fiscal sustainability, to improve the external balance, and to enhance future growth prospects. Can pension reforms help economies cope with those challenges? It is clear how pension reforms that increase the retirement age or cut pension benefits can improve an economy's fiscal balance; yet, their long-term effects on output and the current account are less evident.

We use an overlapping-generations model of a small open economy with a pay-as-you-go pension system to illustrate the long-term effects of "parametric" pension reforms on output and the current account. Conditional on reforms achieving similar fiscal targets, we show the presence of a policy trade-off. Pension reforms that increase the retirement age have a large positive effect on output, but a small (and often negative) effect on the current account. In contrast, reforms that cut pension benefits have a large positive effect on the current account balance but reduce output. We also show that some mixed pension reforms, which extend the working life *and* cut pension benefits, can simultaneously increase output and improve the current account balance. We conclude that increasing the size of the current account effect can only be achieved through a reduction in the size of the output effect, and that this entails larger pension benefit cuts and shorter extensions of the working life.

The effects associated with the two polar pension reforms can be explained as follows. A reform that only increases the retirement age has a large positive effect on the labor supply, attracts capital inflows, and boosts domestic investment and output. The increase in domestic investment, in line with output, is typically combined with a reduction in saving as a share of output—households' response to higher income earnings at the end of their (extended) working lives—resulting in a deterioration of the current account. In contrast, a reform that only cuts pension benefits reduces the effective return on work effort, triggering a reduction in the labor supply, investment, and output. In response to lower retirement income, households save more during their working lives, causing an improvement in the current account balance.

In the literature, pension reform is traditionally viewed as a policy instrument to achieve fiscal objectives like limiting expenditure pressures related to population aging (Lindbeck and Persson 2003).¹ This narrow focus of pension reform analysis contrasts sharply with policy analysis in open economy macroeconomics, where (monetary, fiscal and exchange

¹ A broader set of policy objectives is discussed in relation to pension reforms from pay-as-you-go to fully funded systems. In this case, policy objectives include the development of domestic stock and capital markets, and the increase in domestic saving and growth rates.

rate) policies are assessed in terms of their capacity to achieve both internal and external macroeconomic objectives.

The question whether pension reforms can help individual countries achieve external sector objectives—such as improving the country's current account balance or the sustainability of its external debt—is highly relevant. In this connection, our contribution to the literature is to uncover the presence of a tradeoff between a domestic objective (output) and an external objective (current account balance), and to identify the type of reform that is more effective in achieving each of these objectives.

Related studies show the effects of non-synchronized population aging and pension reform on fiscal sustainability, output, and international capital flows using models of the world economy with multiple regional blocs (large economies); examples include Attanasio and others (2007), Krueger and Ludwig (2007), Fehr and others (2008), Borsch-Supan and others (2006), and Domeij and Floden (2006). These studies are important to understand the dynamics of international capital flows. Unlike this paper, however, they do not evaluate the effects of pension reforms at the level of individual countries and under the "small open economy" assumption. Such evaluation is appropriate for two reasons: first, pension systems and reforms are designed by individual countries and not coordinated internationally; and second, all countries other than the three largest have economies that can be reasonably considered small and open, as argued by Corden (1994).

Other related studies evaluate the effects of pension reforms in specific countries using models of small open economies: Huang and others (USA) (1997), Beetsma and others (Netherlands) (2003), Guest (Australia) (2006), and Catalán and others (Cyprus) (2010a). These studies, however, do not show the presence of a tradeoff between the output and current account effects of pension reforms.

II. THE MODEL

The economy is populated by overlapping generations of finitely-lived households, atomistic firms, and an infinitely-lived government. Households consume and accumulate assets during their lifetime, work during their youth, and retire when old. Firms produce the single good in the model using labor and capital. The government collects income, consumption, and payroll taxes to finance government expenditures and pension benefits, and to redeem the initial government debt. Households and the government can borrow funds from (or invest funds in) international capital markets at prevailing interest rates. The model incorporates

population and labor-augmenting productivity growth and is presented in stationary form.²

Households: The utility of a household born at time *t* is determined by consumption (*c*) and leisure (*l*), and is given by

$$U_{t} = \sum_{s=1}^{T_{t}+T_{t}^{R}} \beta^{s-1} \cdot \left\{ \log(c_{t+s-1}^{s}) + \gamma \cdot \log(l_{t+s-1}^{s}) \right\},$$
(1)

where the household's life is characterized by two phases: a working life lasting T_t years and retirement lasting T_t^R years. The household is endowed with a fixed number of hours per year, normalized to one, and distributed between work (*n*) and leisure (*l*): $l_{t+s-1}^s = 1 - n_{t+s-1}^s$, where $n_{t+s-1}^s = 0$ during retirement.

A household accumulates assets (A) according to the following budget constraint:

$$(1+\xi) \cdot A_{t+s}^{s+1} = [1+r_{t+s-1} \cdot (1-\tau_{t+s-1}^{I})] \cdot A_{t+s-1}^{s} + H_{t+s-1}^{s} - (1+\tau_{t+s-1}^{c}) \cdot c_{t+s-1}^{s},$$
(2)

where $H_{t+s-1}^s = (1 - \tau_{t+s-1} - \tau_{t+s-1}^I) \cdot W_{t+s-1} \cdot e^s \cdot n_{t+s-1}^s$ represents net wage income during the working phase, and $H_{t+s-1}^s = b_{t+s-1}^s$ is the pension benefit during retirement. ξ denotes the annual rate of labor-augmenting technological progress. The household takes as given the payroll (τ), income (τ^I), and consumption (τ^c) tax rates, and the interest (r) and wage (W) rates.³ During the household's working life, next year's assets are determined by adding savings to this year's assets; savings are computed as the sum of net return on assets plus net wage income, minus gross consumption. The household's labor productivity per hour varies

household's age-specific labor effort (n^s) are not adjusted.

³ For simplicity, the income taxes levied on labor income and asset earnings are assumed to be the same. Pension benefits are assumed to be below the minimum taxable income level.

² To transform the model, all aggregate variables except labor are adjusted for labor-augmenting technological progress (ξ) and population size(P). For example, denote \hat{Y} the aggregate output before the transformation; the transformed output Y is given by: $Y_t = \frac{\hat{Y}_t}{(1+\xi)^t \cdot P_t}$. Aggregate labor (N) is adjusted for population size $N_t = \frac{\hat{N}_t}{P_t}$. Household's age-specific consumption and asset holdings (c^s and A^s), and the wage rate (W), are adjusted for technological progress: $W_t = \frac{\hat{W}_t}{(1+\xi)^t}$; $c_t^s = \frac{\hat{C}_t^s}{(1+\xi)^t}$; $A_t^s = \frac{\hat{A}_t^s}{(1+\xi)^t}$. The interest rate (r) and

with age, according to an exogenous skill premium (e^s) defined as the relative productivity of an *s*-year old household to that of a one-year old (unskilled) household.

At retirement, the first pension benefit is calculated based on average past wage earnings, indexed by the economy-wide wage growth; in stationary form, it is given by:

 $b_{t+T_t}^{T_t+1} = \psi \cdot \frac{1}{T_t} \cdot \sum_{j=1}^{t_t} W_{t+j-1} \cdot e^j \cdot n_{t+j-1}^j$, where (ψ) stands for the replacement parameter. Subsequent

pension benefits are indexed annually with wage growth: $b_{t+s-1}^s = b_{t+T_t}^{T_t+1}$, $s = T_t + 2, ..., T_t + T_t^{R_t+4}$. The pension benefit formula affects labor incentives: forward-looking households internalize that higher wage earnings during the working life increase pension benefits—and thus utility—during retirement.

The household's optimization problem is to choose the sequence of consumption, leisure, and asset holdings $\{c_{t+s-1}^{s}, l_{t+s-1}^{s}, A_{t+s-1}^{s}\}_{s=1}^{T_{t}+T_{t}^{R}}$ to maximize its utility (1) subject to the budget constraint (2), the pension benefit formula, and the no intergenerational bequest or inheritance condition $A_{t}^{1} = A_{t}^{T_{t}+T_{t}^{R}+1} = 0$ (for details, see the appendix). Given a total population (P_{t}) and a population of age s (P_{t}^{s}) in year t, aggregate household effective labor supply (N_{t}^{h}), assets (A_{t}^{h}), and consumption (C_{t}^{h}), are respectively given by:

$$N_{t}^{h} = \sum_{s=1}^{T_{t}} e^{s} \cdot n_{t}^{s} \cdot \frac{P_{t}^{s}}{P_{t}}; A_{t}^{h} = \sum_{s=1}^{T_{t}+T_{t}^{R}} A_{t}^{s} \cdot \frac{P_{t}^{s}}{P_{t}}; \text{ and } C_{t}^{h} = \sum_{s=1}^{T_{t}+T_{t}^{R}} c_{t}^{s} \cdot \frac{P_{t}^{s}}{P_{t}}.$$

Firms: Firms maximize profits net of capital depreciation, Π_t^f , using a constant-returns-toscale production function with labor-augmenting technological progress $\Pi_t^f = Z \cdot (K_t^f)^{\alpha} \cdot (N_t^f)^{1-\alpha} - (r_t + \delta) \cdot K_t^f - W_t \cdot N_t^f$, where Z denotes the level of total factor productivity, α is the share of capital in domestic output, K_t^f denotes capital, and δ is the rate of capital depreciation. Both output and factor markets are perfectly competitive; and firms face given wages (W_t) and rental rates (r_t) . The first order conditions require that W_t and $r_t + \delta$ equal, respectively, the marginal product of labor and of capital:

⁴ In an economy with balanced growth, non-stationary transformed wages grow at the rate of labor-augmenting technological progress (see footnote 2).

$$W_t = \mathbf{Z} \cdot (1 - \alpha) \cdot \left(\frac{K_t^f}{N_t^t}\right)^{\alpha}; \ r_t + \delta = \mathbf{Z} \cdot \alpha \cdot \left(\frac{K_t^f}{N_t^t}\right)^{-(1 - \alpha)}$$

Government: The government collects payroll, income, and consumption taxes from households. Tax revenues are used to finance (exogenous) public consumption (G) and pension benefits, and to redeem government debt (D), implying the following budget constraint:

$$D_{t+1} \cdot (1+\xi) \cdot \frac{P_{t+1}}{P_t} = (1+r_t) \cdot D_t + [G_t - \tau_t^I \cdot (r_t \cdot A_t^h + W_t \cdot N_t^h) - \tau_t^c \cdot C_t^h] + \sum_{s=T_t+1}^{T_t + T_t^R} b_t^s \cdot \frac{P_t^s}{P_t} - \tau_t \cdot W_t \cdot N_t^h,$$

where the non-pension primary deficit (in square brackets), and the pension system's deficit (last two terms) are shown separately.

Equilibrium: Given initial conditions and an exogenous path of international interest rates, an equilibrium is defined as a set of allocations for households and firms, prices, and government variables, that simultaneously place all households and firms on their maximizing paths, establish the solvency of the government, and clear all markets. The market-clearing conditions and the economy's aggregate flow constraint are respectively

given by:
$$N_t = N_t^f = \sum_{s=1}^{T_t} \left(e^s \cdot n^s \cdot \frac{P_t^s}{P_t} \right), \quad K_t^f + D_t + A_t^* = A_t^h = \sum_{s=1}^{T_t + T_t^R} \left(A_t^s \cdot \frac{P_t^s}{P_t} \right), \text{ and}$$

$$A_{t+1}^* \cdot (1+\xi) \cdot \frac{P_{t+1}}{P_t} - (1+r_t) \cdot A_t^* = Y_t - C_t - G_t - \left[K_{t+1} \cdot (1+\xi) \cdot \frac{P_{t+1}}{P_t} - (1-\delta) \cdot K_t \right]^{.5}$$
 In the last expression,

the left-hand side is the current account balance, where A^* denotes the economy's net foreign assets, and $Y_t = Y_t^f$ and $C_t = C_t^h$ are the equilibrium aggregate output and consumption, respectively.

$$A_{t+1}^{h} \cdot (1+\xi) \cdot \frac{P_{t+1}}{P_{t}} = [1+r_{t} \cdot (1-\tau_{t}^{I})] \cdot A_{t}^{h} + (1-\tau_{t}^{I}-\tau_{t}) \cdot W_{t} \cdot N_{t}^{h} + \sum_{s=T_{t}+1}^{T_{t}+T_{t}^{R}} b_{t+T_{t}+1-s}^{T_{t}+1} \cdot \frac{P_{t}^{s}}{P_{t}} - (1+\tau_{t}^{c}) \cdot C_{t}^{h}.$$

⁵ The economy's aggregate flow constraint is obtained from the aggregate constraint of the household sector, the first-order conditions of firms, the market equilibrium conditions, and the government budget constraint. The aggregate constraint of the household sector at time t is given by

Balanced Growth Equilibrium: It is defined assuming constant population growth rate (p), labor augmenting technological progress (ξ) , working life $(T_t = T)$, retirement period $(T_t^R = T^R)$, and a fiscal policy characterized by constant tax rates and unchanged public expenditure and debt-to-output ratios. This equilibrium is expressed as a steady state solution in terms of *detrended* variables in the stationary-transformed model.

Parameter Values: Table 1 shows our choice of parameter values. Parameter values corresponding to the utility and production functions (α and β), and the rate of depreciation (δ) are standard in the literature. The leisure parameter (γ) is set so that the fraction of time worked by a representative household is 0.293.⁶ The values for the rate of labor-augmenting technological progress (ξ) and population growth (p) match those observed historically in Spain.⁷ The international interest rate (r) and the discount factor (β) jointly determine the economy's current account and net foreign investment position; we set them to obtain a current account deficit of 3 percent of GDP (output) in the baseline (Table 2). The choice of parameter values related to fiscal and pension systems is guided by historical observations from Spain.⁸ Also based on Spanish data, the working life ($T_t = T$) and retirement period ($T_t^R = T^R$) are set so that households enter the labor force when they are 23 years old, retire at age 63, and die at 81 years old. The labor skill profile (e^s) is hump-shaped, as reported by Hansen (1993).⁹ Unless otherwise indicated, the qualitative results presented below are robust to plausible changes in parameter values.

⁶ Assuming that an individual sleeps/rests 9 hours per day, the leisure-work decision is made for the remaining 15 hours (105 weekly hours). Assume that the individual works 40 hours per week (38.1 percent of non-sleep time). Adjusting the fraction of time worked by a labor force participation rate of 77 percent yields 0.293.

⁷ See Catalán and others (2010b).

⁸ Specifically, following Catalán and others (2010b), the income tax rate (τ^{T}) is set to match Spain's direct tax revenues and other current revenues as percentage of GDP (average 1994-2004). The value of the payroll tax rate (τ) and the replacement parameter (ψ) are set to obtain social security revenue equal to 9.5 percent of GDP and pension spending equal to 8 percent of GDP. The government debt corresponds roughly to the value projected for Spain at mid-2012 (IMF).

⁹ Skills are low at the beginning of the working life, peak at 50 years of age at twice the initial level, and then gradually decline to reach an end-of-working-life level that is 5 percent lower than at the peak.

III. RESULTS

We simulate baseline and pension reform scenarios to illustrate the main qualitative results. In the pension reform scenarios discussed below, the income and payroll tax rates remain unchanged and the consumption tax rate adjusts endogenously to satisfy the government's budget constraint.

A. Baseline

Households' wage income profile by age is hump-shaped as households exert labor effort in tandem with their productive skills (Figure 1). Household income falls at retirement because the pension benefit only partially replaces the wage income; disposable income also falls at retirement despite the wage income being taxed and the pension benefit being tax-exempt. Households choose a rising lifetime consumption profile, which is jointly determined by their subjective rate of time preference and the interest rate. Households incur debt at the beginning of their working lives and accumulate assets during their mid-lives to supplement pension income and boost consumption during retirement.

B. Reform I: Increasing the Retirement Age

The reform increases the retirement age from 63 to 66 years. Household's disposable income is affected by four different forces, ordered by quantitative importance (Figure 2). First, as wage income is higher than the pension benefit, the reform boosts household's income at the end of the working life (ages 63-65). Second, lower consumption taxes that result from the pension reform further boost disposable income. Third, as households are forced to delay retirement for three years, they substitute leisure inter-temporally, working fewer hours than in the baseline when they are aged 23-62 (Figure 3). This effect reduces disposable income and its size is determined by the inter-temporal elasticity of substitution as well as the intra-temporal elasticity of labor supply; this effect is small relative to the other effects noted above even when the labor supply is highly elastic.¹⁰ Fourth, the pension benefit at retirement

aggregate effective labor supply elasticity is $\frac{\partial N^h}{N^h} \cdot \frac{\tilde{W}}{\partial \tilde{W}} = \sum_{s=1}^T e^s \cdot (1-n^s) \cdot \frac{P^s}{P}$ and is greater than 2 given our

¹⁰ The effect would be even smaller under a less elastic labor supply. In this paper, a single parameter (γ) determines the preference for leisure and is calibrated to match the fraction of time worked by a household. This is standard in the literature—see, for example, Miles (1999)—but it implies a labor supply elasticity that is unrealistically high. To see why, ignore the second term in the right-hand side of the household's consumption-leisure decision (Table A1 in the appendix)—which increases the elasticity—and hold consumption constant: the wage elasticity of leisure at age s is unitary and the elasticity of work effort is given by $\frac{1-n^s}{n}$. The

parameterization, where $\tilde{W} = W \cdot (1 - \tau - \tau') / (1 + \tau')$ is the tax-adjusted wage rate. Alternatively, a Frisch-type utility function would provide more flexibility to simultaneously match a lower elasticity and the hours worked; however, it would not affect the main (qualitative) result of this paper: the existence of a tradeoff between the output and current account effects of pension reform.

falls as the extension of the working life lowers the household's average-productivity (due to the hump-shaped labor skill profile). This effect is also small because the skill premium declines slowly with age. On balance, the reform increases household's lifetime disposable income, and hence, consumption.

Compared to the baseline, the higher disposable income at the end of the working life and a shorter retirement period induce households to reduce their saving during most of their working lives (ages 40-62); note that this effect is only partially offset by the effect of lower pension benefits, which encourages households to save more. As a result of higher consumption and a lower pension benefit, household saving also falls during retirement.

The macroeconomic effects of the reform reflect households' optimal responses and aggregation across population cohorts (Table 2). In the economy's real sector, the decline in old-age dependency unambiguously boosts aggregate labor: the contribution of a larger number of working cohorts to aggregate labor more than offsets the aggregate impact of households' inter-temporal leisure substitution effects noted above. The increase in aggregate labor boosts domestic capital investment and output in per capita terms.

On the fiscal front, the reform provides the space needed to reduce the consumption tax rate: a larger number of working cohorts implies higher payroll tax revenue collection and lower aggregate pension spending; the latter is further strengthened by the reduction in individual pension benefits.

The current account balance declines relative to the baseline, reflecting lower aggregate saving as a fraction of output and domestic investment that rises in line with output. Aggregate saving falls as a fraction of output because the saving decline of cohorts aged 40-62 and 66-80 dominates the increase in saving by other cohorts. Investment remains unchanged as a fraction of output: more domestic investment and capital are needed to complement the increase in labor supply while keeping the marginal productivity of capital unchanged in the presence of free international financial flows. In terms of the tradeoff between output and the current account, this reform increases output but worsens the current account balance (point A in Figure 4).

C. Reform II: Cutting Pension Benefits

The government cuts pension benefits by reducing the value of the replacement parameter (ψ) . This parametric change is calibrated to deliver the same pension expenditure reduction (as percentage of GDP) obtained with the reform that increased the retirement age, discussed

above. From a household's standpoint, the reform reduces the effective return on work effort;¹¹ hence, the labor effort exerted by households aged 40-62 declines (Figure 3)—the intensity of this decline depends on the labor supply elasticity, which in this paper is high (see footnote 10). Households supplement their lower pension income, to smooth lifetime consumption, by increasing saving during their working lives. Faster asset accumulation during the working life and lower consumption taxes increase disposable income and allow households to increase lifetime consumption (Figure 2).

The reform's macroeconomic effects are as follows (Table 2). In the real sector, aggregate labor, capital, and output decline in per capita terms. The aggregate labor effect is negative because the number of working cohorts remains unchanged and households reduce the number of hours worked during their working lives; also, a fall in domestic investment is needed to keep the marginal productivity of capital in line with the international interest rate.

On the fiscal front, aggregate pension spending falls as individual pension benefits are lower, allowing a reduction in the consumption tax rate. In the external sector, the current account improves, reflecting a large increase in saving as a share of output and a decline in domestic investment in line with output. In sum, this reform delivers a negative change in output and a positive change in the current account balance (point D in Figure 4).

D. The Long-Run Tradeoff between Output and the Current Account

We have discussed the effects of two polar pension reforms—which either increase the retirement age or cut pension benefits (points A and D in Figure 4). We now consider the effects of mixed reforms that simultaneously cut pension benefits and increase the retirement age while achieving the same expenditure reductions as the polar cases.

In Figure 4, point B corresponds to a reform that increases the retirement age by two years and cuts pension benefits, while point C corresponds to a reform that increases the retirement age by only one year but relies more on pension benefit cuts.

disincentive effect is captured by the term $W_{t+s-1} \cdot e^s \cdot \frac{\psi}{T_t} \cdot \beta^{T_t+1-s} \cdot V_b(A_{t+T_t}^{T_t+1}, b_{t+T_t}^{T_t+1})$ in the household's consumptionleisure decision (Table A1 in the appendix).

¹¹ In standard inter-temporal models of household optimization, households increase work effort in anticipation of future (exogenous) income losses. In sharp contrast, in this model the reform reduces the return on labor effort through the pension benefit formula: a smaller replacement parameter implies that higher wage earnings in the working life result in smaller increases in the pension benefit received during retirement. The labor

The A-D curve makes evident our main conclusion: for a given expenditure reduction target (as percentage of GDP), there is a long-run tradeoff between the output and current account effects associated with pension reforms. Increasing the size of the current account effect can only be achieved through a reduction in the size of the output effect—through left-upward movements along the A-D curve—and this entails larger pension benefit cuts and shorter extensions of the working life.¹²

We find that, in general, both cuts in pension benefits and extensions of the working life must be part of a reform that simultaneously seeks to boost output and the current account. In some cases, however, both objectives can be achieved through reforms that only increase the retirement age. Consider, as an example, an economy with a large pension system where initially $\psi = 0.4$ and pension expenditure is 9.7 percent of GDP. The A'-D' curve shows the effects of pension reforms in such an economy. Three observations are noteworthy. First, the tradeoff between output and the current account remains present. Second, the A'-D' curve lies above the A-D curve, implying that the larger is the initial size of the pension system, the larger are the effects of reforms. Third, point A' lies in the positive quadrant, showing that a reform that only increases the retirement age can boost both output and, to a lesser extent, the current account.¹³

Figure 4 also shows that the output-current account tradeoff remains robust if the fiscal objective consists in achieving a given (consumption) tax rate target, as illustrated by the A-E curve, rather than a pension spending-to-GDP target.

To conclude, we have established the robust presence of a *long-term* tradeoff between the output and current account effects of pension reforms, conditional on reforms achieving similar fiscal targets.¹⁴

¹² In regard to the welfare effects of pension reforms, we find that for a given fiscal target, reforms that increase the retirement age yield smaller welfare gains than reforms that cut pension benefits. In the context of our model, the welfare gains of pension reforms relative to the baseline can be measured using (stationary-transformed) household utility levels. Normalizing household lifetime utility to 1 in the baseline, the reforms along the A-D curve yield the following gains: reform A: 1.0035 (a 0.35 percent gain relative to the baseline); reform B: 1.0046; reform C: 1.0058; and reform D: 1.0069.

¹³ In this case, the current account improves because aggregate saving (as a share of output) increases. When the pension system is large, pension benefits are generous relative to wage earnings prior to retirement. Hence, an increase in the retirement age results in a smaller increase in disposable income at the end of the working life. In response, a smaller number of household cohorts reduce saving during their working lives.

¹⁴ An issue that deserves further attention but exceeds the scope of this paper is whether such tradeoff could also be present in the short- and medium terms, and if so, under what conditions. At such horizons, the presence of the tradeoff could be sensitive to grandfathering arrangements and the sequencing of fiscal targets.

Symbol	Definition	Value	Symbol	Definition	Value
α	Share of capital	0.300	τ	Social security payroll tax rate	0.142
γ	Leisure preference	3.050	$ au^{I}$	Capital-income tax rate	0.130
β	Discount factor	0.985	G/Y	Government consumption (fraction of total output)	0.230
δ	Depreciation rate	0.060	D/Y	Government debt (fraction of total output)	0.750
ک	Rate of labor-augmenting technological progress	0.015	Т	Work life (years)	40.000
р	Rate of population growth	0.008	T^{R}	Retirement life (years)	18.000
r	Interest rate	0.050	Ψ	Replacement ratio	0.330

Table 1. Baseline Parameter Values



1/ It indicates natural age. A household's age in the model corresponds to the number of years since it entered the labor force (age 1 in the model = natural age 23).



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								% of GDP
Real sector								
Capital	3.000		3.050		2.984		2.988	
Effective Labor	1.217		1.237		1.211		1.212	
Output	1.000	100.0	1.017	100.0	0.995	100.0	0.996	100.0
Net Factor Income	-0.064	-6.4	-0.067	-6.5	-0.043	-4.4	-0.048	-4.8
Gross National Income	0.936	93.6	0.950	93.5	0.951	95.6	0.948	95.2
Private Consumption	0.486	48.6	0.493	48.5	0.495	49.7	0.493	49.5
Government Consumption	0.230	23.0	0.234	23.0	0.229	23.0	0.229	23.0
Investment	0.249	24.9	0.253	24.9	0.248	24.9	0.248	24.9
Current Account	-0.030	-3.0	-0.031	-3.0	-0.020	-2.0	-0.022	-2.2
National Saving	0.220	22.0	0.223	21.9	0.228	22.9	0.226	22.7
External sector								
Net Exports	0.021	2.1	0.036	3.5	0.023	2.3	0.026	2.6
Net Factor Income	-0.064	-6.4	-0.067	-6.5	-0.043	-4.4	-0.048	-4.8
Current Account	-0.030	-3.0	-0.031	-3.0	-0.020	-2.0	-0.022	-2.2
Financial Account	0.030	3.0	0.031	3.0	0.020	2.0	0.022	2.2
Fiscal sector								
Consumption Tax Rate (percent)	27.1		23.4		22.3		23.4	
Social security								
Payroll Tax Revenue	0.095	9.5	0.097	9.5	0.095	9.5	0.095	9.5
Pension Expenditure	0.080	8.0	0.062	6.1	0.061	6.1	0.066	6.6
Balance	0.015	1.5	0.034	3.4	0.034	3.4	0.029	2.9
Non-social security								
Income tax revenue	0.103	10.3	0.105	10.3	0.105	10.6	0.105	10.5
Consumption tax revenue	0.132	13.2	0.115	11.4	0.110	11.1	0.115	11.6
Revenue non-social security	0.235	23.5	0.220	21.6	0.215	21.6	0.220	22.1
Primary expenditure non-social security	0.230	23.0	0.234	23.0	0.229	23.0	0.229	23.0
Interest payments	0.037	3.7	0.038	3.7	0.037	3.8	0.037	3.7
Balance non-social security	-0.033	-3.3	-0.052	-5.1	-0.051	-5.1	-0.046	-4.7
Overall								
Revenue	0.330	33.0	0.317	31.2	0.310	31.2	0.315	31.6
Expenditure	0.347	34.7	0.334	32.9	0.327	32.9	0.332	33.3
Balance	-0.017	-1.7	-0.018	-1.7	-0.017	-1.7	-0.017	-1.7
Asset decomposition								
Assets	2.471	247.1	2.481	244.1	2.862	287.7	2.769	278.1
Capital	3.000	300.0	3.050	300.0	2.984	300.0	2.988	300.0
Government Debt	0.750	75.0	0.762	75.0	0.746	75.0	0.747	75.0
Net Foreign Assets	-1.279	-127.9	-1.331	-130.9	-0.868	-87.3	-0.965	-96.9
Pension system								
Pension benefit at retirement (percent of output per capita)	30.4		28.9		23.3		25.0	
Replacement rate (percent of average wage income)	33.0		33.0		25.4		27.2	
Replacement rate (percent of last wage income)	43.3		56.9		35.1		37.1	

Table 2. Results (Unless otherwise indicated, all variables are expressed as a fraction of de-trended output in the baseline)

Notes: in the baseline scenario, primary balance (2 percent of GDP) – interest payments (3.7 percent of GDP) = overall balance (-1.7 percent of GDP), where the primary balance stabilizes the government debt at 75 percent of GDP. Consistent with the government's budget constraint, the debt-stabilizing primary balance is given by $\frac{D}{Y} \cdot \left\{ r - \left[(1 + \xi) \cdot (1 + p) - 1 \right] \right\} = 0.75 \times \left\{ 0.05 - [1.015 \times 1.008 - 1] \right\} = 0.02.$

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Appendix. Household's Optimization Problem

The household's problem can be expressed as a sequence of two dynamic optimization problems, as follows:

$$\underset{\left\{c_{t+s-1}^{s}, l_{t+s-1}^{s}, A_{t+s}^{s+1}\right\}_{s=1}^{T_{t}}}{Max} \sum_{s=1}^{T_{t}} \beta^{s-1} \cdot \left\{ \log(c_{t+s-1}^{s}) + \gamma \cdot \log(l_{t+s-1}^{s}) \right\} + \beta^{T_{t}} \cdot V(A_{t+T_{t}}^{T_{t}+1}, b_{t+T_{t}}^{T_{t}+1})$$

subject to (2); $b_{t+T_t}^{T_t+1} = \psi \cdot \frac{1}{T_t} \cdot \sum_{j=1}^{T_t} W_{t+j-1} \cdot e^j \cdot n_{t+j-1}^j$; $b_{t+s-1}^s = b_{t+T_t}^{T_t+1}$, $s = T_t + 2, \dots, T_t + T_t^R$; and

 $A_t^1 = A_t^{T_t+T_t^R+1} = 0$. $V(A_{t+T_t}^{T_t+1}, b_{t+T_t}^{T_t+1})$ is the household's value function or discounted indirect utility when it retires at time $t + T_t$ having reached the age of $T_t + 1$ years. Upon retirement, the household's optimization problem can be expressed recursively, and a closed-form solution for the value function (V) follows from the log utility assumption.¹⁵ In the rest of the appendix, we first show the derivation of the value function; then we show the first order conditions of the household's optimization problem (Table A1).

Value Function at Retirement: The value function is the solution of the following problem:

$$V(A_{t+T_{t}}^{T_{t}+1}, b_{t+T_{t}}^{T_{t}+1}) = \underset{\{c_{t+s-1}^{s}, A_{t+s}^{s+1}\}_{s=T_{t}+1}^{T_{t}+T_{t}^{R}}}{Max} \sum_{s=T_{t}+1}^{T_{t}+T_{t}^{R}} \beta^{s-1} \cdot \log(c_{t+s-1}^{s}) \text{ subject to (2); } b_{t+T_{t}}^{T_{t}+1} = \psi \cdot \frac{1}{T_{t}} \cdot \sum_{j=1}^{T_{t}} W_{t+j-1} \cdot e^{j} \cdot n_{t+j-1}^{j};$$

$$b_{t+s-1}^{s} = b_{t+T_{t}}^{T_{t}+1}, s = T_{t} + 2, \dots, T_{t} + T_{t}^{R}; A_{t}^{T_{t}+T_{t}^{R}+1} = 0; \text{ and given } A_{t+T_{t}}^{T_{t}+1} \text{ and } b_{t+T_{t}}^{T_{t}+1}. \text{ The household's asset}$$

holdings at retirement $(A_{t+T_{t}}^{T_{t}+1}), \text{ and the annual pension benefit } (b_{t+T_{t}}^{T_{t}+1}) \text{ are given, as they are}$
determined by household's past decisions. We obtain the value function by backward
induction, starting with the household's problem in its last year of life.

1. The household's problem at date $t + T_t + T_t^R - 1$ (household's age is $s = T_t + T_t^R$) is given by

$$V\left(A_{t+T_{t}+T_{t}^{R}-1}^{T_{t}+T_{t}^{R}}, b_{t+T_{t}}^{T_{t}+1}\right) = \max_{A_{t+T_{t}+T_{t}^{R}}^{T_{t}+T_{t}^{R}+1}} \log\left\{\left(1+\tilde{r}_{t+T_{t}+T_{t}^{R}-1}\right) \cdot A_{t+T_{t}+T_{t}^{R}-1}^{T_{t}+T_{t}^{R}} + b_{t+T_{t}}^{T_{t}+1} - \left(1+\xi\right) \cdot A_{t+T_{t}+T_{t}^{R}}^{T_{t}+T_{t}^{R}+1}\right\}$$

¹⁵ Note that the value function V(.) depends also on future interest rates and income tax rates.

The household consumes all its remaining assets in its last period of life, as it leaves no bequests and the no-Ponzi condition $(A_{t+T_t+T_t^R}^{T_t+T_t^R} = 0)$ is satisfied. Thus, the solution is given by

$$V\left(A_{t+T_{t}+T_{t}^{R}-1}^{T_{t}+T_{t}^{R}}, b_{t+T_{t}}^{T_{t}+1}\right) = \log\left\{\left(1+\tilde{r}_{t+T_{t}+T_{t}^{R}-1}\right) \cdot A_{t+T_{t}+T_{t}^{R}-1}^{T_{t}+T_{t}^{R}} + b_{t+T_{t}}^{T_{t}+1}\right\}$$

2. The household's problem at date $t + T_t + T_t^R - 2$ (household's age is $T_t + T_t^R - 1$) is given by

$$V\left(A_{t+T_{t}+T_{t}^{R}-2}^{T_{t}+T_{t}^{R}-1}, b_{t+T_{t}}^{T_{t}+1}\right) = \max_{\substack{A_{t+T_{t}}^{T_{t}+T_{t}^{R}} \\ t+T_{t}+T_{t}^{R}-1}} \log\left\{\left(1+\tilde{r}_{t+T_{t}+T_{t}^{R}-2}\right) \cdot A_{t+T_{t}+T_{t}^{R}-2}^{T_{t}+T_{t}^{R}-1} + b_{t+T_{t}}^{T_{t}+1} - \left(1+\xi\right) \cdot A_{t+T_{t}+T_{t}^{R}-1}^{T_{t}+T_{t}^{R}}\right\} + \beta \cdot V\left(A_{t+T_{t}+T_{t}^{R}-1}^{T_{t}+T_{t}^{R}-1}, b_{t+T_{t}}^{T_{t}+1}\right).$$

Plug the solution of $V\left(A_{t+T_{t}+T_{t}^{R}-1}^{T_{t}+T_{t}^{R}}, b_{t+T_{t}}^{T_{t}+1}\right)$ found in 1 to obtain the following expression:

$$V\left(A_{t+T_{t}+T_{t}^{R}-2}^{T_{t}+T_{t}^{R}-1}, b_{t+T_{t}}^{T_{t}+1}\right) = \underset{A_{t+T_{t}+T_{t}^{R}-1}^{T_{t}+T_{t}^{R}}}{A_{t+T_{t}+T_{t}^{R}-1}^{T_{t}+T_{t}^{R}}} \log\left\{\left(1+\tilde{r}_{t+T_{t}+T_{t}^{R}-2}\right) \cdot A_{t+T_{t}+T_{t}^{R}-2}^{T_{t}+T_{t}^{R}-1} + b_{t+T_{t}}^{T_{t}+1} - \left(1+\xi\right) \cdot A_{t+T_{t}+T_{t}^{R}-1}^{T_{t}+T_{t}^{R}}\right\} + \beta \cdot \log\left\{\left(1+\tilde{r}_{t+T_{t}+T_{t}^{R}-1}\right) \cdot A_{t+T_{t}+T_{t}^{R}-1}^{T_{t}+T_{t}^{R}} + b_{t+T_{t}}^{T_{t}+1}\right\}.$$

Find the first order condition of this optimization problem and solve for $A_{t+T_t+T_t^{R-1}}^{T_t+T_t^{R}}$,

$$A_{t+T_{t}+T_{t}^{R}-1}^{T_{t}+T_{t}^{R}} = \frac{\beta \cdot \prod_{i=1}^{2} \left(1 + \tilde{r}_{t+T_{t}+T_{t}^{R}-i}\right) \cdot A_{t+T_{t}+T_{t}^{R}-2}^{T_{t}+T_{t}^{R}-1} - \left[1 + \xi - \beta \cdot \left(1 + \tilde{r}_{t+T_{t}+T_{t}^{R}-1}\right)\right] \cdot b_{t+T_{t}}^{T_{t}+1}}{\left(1 + \xi\right) \cdot \left(1 + \beta\right) \cdot \left(1 + \tilde{r}_{t+T_{t}+T_{t}^{R}-1}\right)};$$

plug this expression into the value function $V\left(A_{t+T_t+T_t^{R}-2}^{T_t+T_t^{R}-1}, b_{t+T_t}^{T_t+1}\right)$ and solve, as follows:

$$V\left(A_{t+T_{t}+T_{t}^{R}-2}^{T_{t}+T_{t}^{R}-1}, b_{t+T_{t}}^{T_{t}+1}\right) = (1+\beta) \cdot \log\left\{\prod_{i=1}^{2} \left(1+\tilde{r}_{t+T_{t}+T_{t}^{R}-i}\right) \cdot A_{t+T_{t}+T_{t}^{R}-2}^{T_{t}+T_{t}^{R}-1} + \left(2+\tilde{r}_{t+T_{t}+T_{t}^{R}-1}+\xi\right) \cdot b_{t+T_{t}}^{T_{t}+1}\right\} - \Omega_{1},$$

where Ω_1 is a constant: $\Omega_1 = \log(1 + \tilde{r}_{t+T_t+T_t^R-1}) + (1+\beta) \cdot \log(1+\beta) + \beta \cdot \log(1+\xi) - \beta \cdot \log(\beta)$.

3. The household's problem at date $t + T_t + T_t^R - 3$ (household's age is $s = T_t + T_t^R - 2$):

$$V\left(A_{t+T_{t}+T_{t}^{R}-3}^{T_{t}+T_{t}^{R}-2}, b_{t+T_{t}}^{T_{t}+1}\right) = \max_{\substack{A_{t}^{T_{t}+T_{t}^{R}-1}\\t+T_{t}+T_{t}+T_{t}^{R}-2}} \log\left\{\left(1+\tilde{r}_{t+T_{t}+T_{t}^{R}-3}\right) \cdot A_{t+T_{t}+T_{t}^{R}-3}^{T_{t}+T_{t}^{R}-2} + b_{t+T_{t}}^{T_{t}+1} - \left(1+\xi\right)A_{t+T_{t}+T_{t}^{R}-2}^{T_{t}+T_{t}^{R}-1}\right\} + \beta \cdot V\left(A_{t+T_{t}+T_{t}^{R}-2}^{T_{t}+T_{t}^{R}-1}, b_{t+T_{t}}^{T_{t}+1}\right).$$

Replacing $V\left(A_{t+T_t+T_t^R-2}^{T_t+T_t^R-1}, b_{t+T_t}^{T_t+1}\right)$ from 2, we can write the previous expression as follows:

$$V\left(A_{t+T_{t}+T_{t}^{R}-3}^{T_{t}+T_{t}^{R}-2}, b_{t+T_{t}}^{T_{t}+1}\right) = \underset{A_{t+T_{t}+T_{t}^{R}-2}^{T_{t}+T_{t}^{R}-1}}{A_{t+T_{t}+T_{t}^{R}-3}^{T_{t}+T_{t}^{R}-2}} \log\left\{\left(1+\tilde{r}_{t+T_{t}+T_{t}^{R}-3}\right) \cdot A_{t+T_{t}+T_{t}^{R}-3}^{T_{t}+T_{t}^{R}-2} + b_{t+T_{t}}^{T_{t}+1} - (1+\xi) \cdot A_{t+T_{t}+T_{t}^{R}-2}^{T_{t}+T_{t}^{R}-1}\right\} + \beta \cdot (1+\beta) \cdot \log\left\{\prod_{i=1}^{2}\left(1+\tilde{r}_{t+T_{t}+T_{t}^{R}-i}\right) \cdot A_{t+T_{t}+T_{t}^{R}-2}^{T_{t}+T_{t}^{R}-1} + \left(2+\tilde{r}_{t+T_{t}+T_{t}^{R}-1}^{T_{t}+1} + \xi\right) \cdot b_{t+T_{t}}^{T_{t}+1}\right\} - \beta \cdot \Omega_{1}.$$

Find the first order condition and solve for $A_{t+T_t+T_t^{R}-1}^{T_t+T_t^{R}-1}$,

$$A_{t+T_{t}+T_{t}^{R}-2}^{T_{t}+T_{t}^{R}-1} = \frac{\beta \cdot (1+\beta) \cdot \prod_{i=1}^{3} \left(1+\tilde{r}_{t+T_{t}+T_{t}^{R}-i}\right) \cdot A_{t+T_{t}+T_{t}^{R}-3}^{T_{t}+T_{t}^{R}-2} + \left[\beta \cdot (1+\beta) \cdot \prod_{i=1}^{2} \left(1+\tilde{r}_{t+T_{t}+T_{t}^{R}-i}\right) - (1+\xi) \cdot \left(2+\tilde{r}_{t+T_{t}+T_{t}^{R}-1}+\xi\right)\right] \cdot b_{t+T_{t}}^{T_{t}+1}}{(1+\xi) \cdot (1+\beta+\beta^{2}) \cdot \prod_{i=1}^{2} \left(1+\tilde{r}_{t+T_{t}+T_{t}^{R}-i}\right)}$$

Plug this previous expression into the value function $V\left(A_{t+T_t+T_t^{R}-3}^{T_t+T_t^{R}-2}, b_{t+T_t}^{T_t+1}\right)$ and solve,

$$V\left(A_{l+T_{t}+T_{t}^{R}-3}^{T_{t}+T_{t}^{R}-2}, b_{l+T_{t}}^{T_{t}+1}\right) = \left(1+\beta+\beta^{2}\right) \cdot \log\left\{\prod_{i=1}^{3}\left(1+\tilde{r}_{l+T_{t}+T_{t}^{R}-i}\right) \cdot A_{l+T_{t}+T_{t}^{R}-3}^{T_{t}+T_{t}^{R}-2} + \left[\prod_{i=1}^{2}\left(1+\tilde{r}_{l+T_{t}+T_{t}^{R}-i}\right) + \left(2+\xi+\tilde{r}_{l+T_{t}+T_{t}^{R}-1}\right)\right] \cdot b_{l+T_{t}}^{T_{t}+1}\right\} - \Omega_{2},$$

where Ω_2 is a constant: $\Omega_2 = \log(1 + \tilde{r}_{t+T_t + T_t^R - 1}) + \log(1 + \tilde{r}_{t+T_t + T_t^R - 2}) + (1 + \beta + \beta^2) \cdot \log(1 + \beta + \beta^2) + \beta \cdot (1 + \beta) \cdot \log(1 + \beta) \cdot \log(1 + \beta) \cdot \log[\beta \cdot (1 + \beta)] + \beta \cdot \Omega_1.$

4. Repeating the procedure backwards, the value function at date $t + T_t$ (household's age is $T_t + 1$) is given by

$$V\left(A_{t+T_{t}}^{T_{t}+1}, b_{t+T_{t}}^{T_{t}+1}\right) = \left(\sum_{j=1}^{T_{t}^{R}} \beta^{j-1}\right) \cdot \log\left\{A_{t+T_{t}}^{T_{t}+1} \cdot \prod_{i=1}^{T_{t}^{R}} \left(1 + \tilde{r}_{t+T_{t}+T_{t}^{R}-i}\right) + b_{t+T_{t}}^{T_{t}+1} \cdot \left\{\prod_{i=1}^{T_{t}^{R}-1} \left(1 + \tilde{r}_{t+T_{t}+T_{t}^{R}-i}\right) + \left(1 + \xi\right) \cdot \left[1 + \xi + \sum_{j=1}^{T_{t}^{R}-2} \prod_{i=1}^{j} \left(1 + \tilde{r}_{t+T_{t}+T_{t}^{R}-i}\right)\right]\right\}\right\} - \Omega,$$

where Ω is a constant. The derivatives of the value function with respect to changes in asset holdings (V_A) and pension benefits (V_b) are given by

$$V_{A}(.) = \frac{\left(\sum_{j=1}^{T_{t}^{R}} \beta^{j-1}\right) \cdot \prod_{i=1}^{T_{t}^{R}} \left(1 + \tilde{r}_{t+T_{t}+T_{t}^{R}-i}\right)}{A_{t+T_{t}}^{T_{t}+1} \cdot \prod_{i=1}^{T_{t}^{R}} \left(1 + r_{t+T_{t}+T_{t}^{R}-i}\right) + b_{t+T_{t}}^{T_{t}+1} \cdot \left\{\prod_{i=1}^{T_{t}^{R}-1} \left(1 + r_{t+T_{t}+T_{t}^{R}-i}\right) + (1 + \xi) \cdot \left[1 + \xi + \sum_{j=1}^{T_{t}^{R}-2} \prod_{i=1}^{j} \left(1 + \tilde{r}_{t+T_{t}+T_{t}^{R}-i}\right)\right]\right\},$$

$$V_{b}(.) = \frac{\left(\sum_{j=1}^{T_{t}^{R}} \beta^{j-1}\right) \cdot \left\{\prod_{i=1}^{T_{t}^{R}-1} \left(1 + r_{t+T_{t}+T_{t}^{R}-i}\right) + (1 + \xi) \cdot \left[1 + \xi + \sum_{j=1}^{T_{t}^{R}-2} \prod_{i=1}^{j} \left(1 + \tilde{r}_{t+T_{t}+T_{t}^{R}-i}\right)\right]\right\}}{A_{t+T_{t}}^{T_{t}+1} \cdot \prod_{i=1}^{T_{t}^{R}} \left(1 + r_{t+T_{t}+T_{t}^{R}-i}\right) + b_{t+T_{t}}^{T_{t}+1} \cdot \left\{\prod_{i=1}^{T_{t}^{R}-1} \left(1 + r_{t+T_{t}+T_{t}^{R}-i}\right) + (1 + \xi) \cdot \left[1 + \xi + \sum_{j=1}^{T_{t}^{R}-2} \prod_{i=1}^{j} \left(1 + \tilde{r}_{t+T_{t}+T_{t}^{R}-i}\right)\right]\right\}}.$$

 Table A1. Household's Optimization Problem—First Order Conditions

Consumption-Leisure Decision	Consumption-Saving Decision
(Intratemporal condition)	(Intertemporal condition)

Working Age

$$\frac{\gamma}{l_{t+s-1}^{s}} = \frac{W_{t+s-1} \cdot e^{s} \cdot (1 - \tau_{t+s-1} - \tau_{t+s-1}^{I})}{c_{t+s-1}^{s} \cdot (1 + \tau_{t+s-1}^{c})} + W_{t+s-1} \cdot e^{s} \cdot \frac{\psi}{T_{t}} \cdot \beta^{T_{t}+1-s} \cdot V_{b}(A_{t+T_{t}}^{T_{t}+1}, b_{t+T_{t}}^{T_{t}+1}) \qquad \frac{(1 + \xi)}{c_{t+s-1}^{s} \cdot (1 + \tau_{t+s-1}^{c})} = \beta \cdot \frac{[1 + r_{t+s} \cdot (1 - \tau_{t+s}^{I})]}{c_{t+s}^{s+1} \cdot (1 + \tau_{t+s}^{c})}$$

$$(s = T_{t}) \qquad \qquad \frac{\gamma}{l_{t+T_{t}-1}^{T_{t}}} = \frac{W_{t+T_{t}-1} \cdot e^{s} \cdot (1 - \tau_{t+T_{t}-1} - \tau_{t+T_{t}-1}^{T})}{c_{t+T_{t}-1}^{T_{t}} \cdot (1 + \tau_{t+T_{t}-1}^{c})} + W_{t+T_{t}-1} \cdot e^{s} \cdot \frac{\psi}{T_{t}} \cdot \beta \cdot V_{b}(A_{t+T_{t}}^{T_{t+1}}, b_{t+T_{t}}^{T_{t+1}}) \qquad \qquad \frac{(1 + \xi)}{c_{t+T_{t}-1}^{T_{t}} \cdot (1 + \tau_{t+T_{t}}^{c}, b_{t+T_{t}-1}^{T_{t+1}})} = \beta \cdot V_{A}(A_{t+T_{t}}^{T_{t+1}}, b_{t+T_{t}}^{T_{t+1}})$$

Retirement

 $(s = T_t + 1, ..., T_t + T_t^R - 1)$

$$\frac{(1+\xi)}{c_{t+s-1}^{s} \cdot (1+\tau_{t+s-1}^{c})} = \frac{\beta \cdot [1+r_{t+s} \cdot (1-\tau_{t+s}^{l})]}{c_{t+s}^{s+1} \cdot (1+\tau_{t+s}^{c})}$$

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